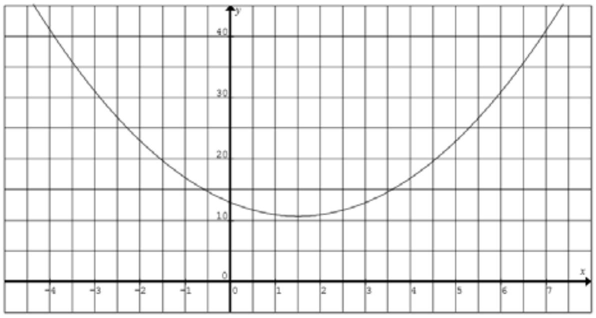
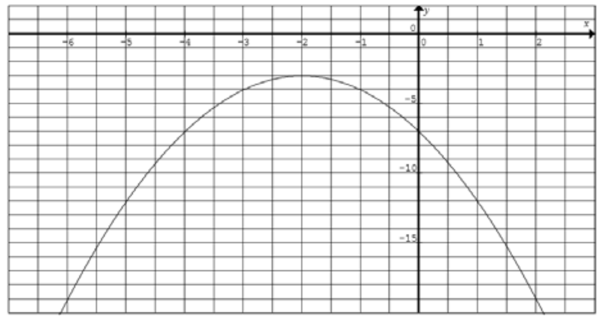
Precalculus 11 **Sec 3.3 – Completing the Square (Part 1)**

The problem with the General Form and the textbook method is twofold:

1. The function is not easily factorable $y = x^2 - 3x + 13$ 	2. The function has no x-intercepts $y = -x^2 - 4x - 7$ 

Completing the Square (CTS) is the second, and perhaps the best, method that addresses these two problems directly

1. It changes any quadratic equation in general form to standard form

General Form		Standard Form
$y = ax^2 + bx + c$	$\Rightarrow$	$y = a(x-p)^2 + q$

2. It makes graphing and finding the x-intercepts a lot easier and faster

*The CTS also "gives birth" to the Quadratic Formula, to help solve equations that cannot be factored, i.e., you get x-intercepts that are fractions or decimals

Let's go back and ensure we understand what is a "perfect square trinomial"

$y = (x+3)(x+3) = (x+3)^2$ $y = x^2 + 6x + 9$ $y = (x+4)(x+4) = (x+4)^2$ $y = x^2 + 8x + 16$ $y = (x-5)(x-5) = (x-5)^2$ $y = x^2 - 10x + 25$	• Notice that the constant term in the trinomial is always a perfect square • Notice that in the trinomial, if you divide the coefficient of the second term by 2 and square the #, you get the constant term • Notice that the constant term in the trinomial is always a positive value because of the squaring process

Basically, what we're doing is <u>using CTS to create a "perfect square trinomial"</u> to transform any quadratic equation from general form to standard form

$y = ax^2 + bx + c$	
$y = x^2 + \frac{b}{a}x + \frac{c}{a}$	• Divide entire right side by 'a'
$y = \left(x^2 + \frac{b}{a}x\right) + \frac{c}{a}$	• Put a set of bracket around the first two terms
$y = \left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\right) + \frac{c}{a} + \left(-\left(\frac{b}{2a}\right)^2\right)$	• Divide $\frac{b}{a}$ by 2, square the whole term, then add its opposite to the constant term $\frac{c}{a}$
$y = \left(x + \frac{b}{2a}\right)^2 + \frac{c}{a} + \left(-\frac{b^2}{4a^2}\right)$	• The trinomial inside the brackets is a perfect square, so simplify
$y = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$	• Eliminate the denominator, so multiply entire right side by 'a'
$y = a\left(x - \left(-\frac{b}{2a}\right)\right)^2 + c - \frac{b^2}{4a}$	• Recall that $p = -\frac{b}{2a}$
$y = a\left(x - \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$	• Recall $q = c - ap^2$, therefore we have $q = c - a\left(-\frac{b}{2a}\right)^2$ = $q = c - \left(\frac{b^2}{4a}\right)$
$y = a(x-p)^2 + q$	• Equation is now in standard form

Example 1: Given $y = x^2 - 6x + 7$, complete the square and find the vertex, A of S, x & y-intercepts, and the domain & range

$y = x^2 - 6x + 7$	• Put brackets around first two terms
$y = (x^2 - 6x) + 7$	
$y = \left(x^2 - (6)x + \left(\frac{6}{}\right)^2\right) + 7 + (- \quad)$	• Divide 2nd term coefficient by ___, _________ it, then add its _________ on the outside
$y = (x^2 - \quad x + \quad) -$	• You now have a "__________ __________" trinomial
$y = (x - \quad)^2 -$	• Simplify to __________ form

• Vertex = • A of S: $x =$ • $y = (\quad -3)^2 - 2$ $y = (\quad)^2 - 2$ $y = \quad -2 =$ • $\quad = (x-3)^2 - 2$ $\quad = (x-3)^2$ $\pm\sqrt{\quad} = \sqrt{(x-3)^2}$ $\pm\sqrt{\quad} = x-3$ $x = \quad \pm\sqrt{\quad}$ $x = \quad +\sqrt{\quad}$ and $x = \quad -\sqrt{\quad}$ $x =$ and $x =$ Domain: $\{x \mid \quad\}$ Range: $\{y \mid \quad\}$	• Remember the vertex is (p, q) • Remember that A of S is $x = p$ • For y-int, remember x-coordinate = __ • For x-int, remember y-coordinate = __ • Domain is all "x" that satisfy the function • Range is all "y" that satify the function

Example 2: Given $y = x^2 + 10x + 14$, complete the square and find the vertex, A of S, x & y-intercepts, and the domain & range

Example 3: Given $y = 2x^2 - 12x + 11$, complete the square and find the vertex, A of S, x & y-intercepts, and the domain & range

$y = 2x^2 - 12x + 11$ $y = \frac{2}{\ }x^2 - \frac{12}{\ }x + \frac{11}{\ }$ $y = \ x^2 - \ x + \frac{11}{\ }$	• Divide entire right side by __
$y = (x^2 - \ x) + \frac{11}{\ }$	• Put bracket around 1st two terms
$y = \left(x^2 - \ x + \left(\frac{\ }{\ }\right)^2\right) + \frac{11}{\ } + (- \ \ )$	• Divide 2nd term coefficient by __, ______ it, then add its ________ on the outside
$y = (x^2 - \ x + \ \ ) + \frac{11}{\ } -$ $y = \ (x^2 - \ x + \ \ ) + \ \ -$	• Multiply entire right side by __
$y = \ (x - \ \ )^2 -$	• Simplify perfect square trinomial into standard form

Vertex:	A of S: $x =$
y-intercept:	x-intercepts:
Domain: $\{x \mid \qquad \}$	Range: $\{y \mid \qquad \}$